APPROVED
at a meeting of the
Scientific Council
NJSC «Al-Farabi KazNU».
Minutes No.10 dated
May 23, 2022.

The program of the entrance exam for applicants to the PhD for the group of educational programs D092 - «Mathematics and statistics»

1. General provisions.

1. The program was drawn up in accordance with the Order of the Minister of Education and Science of the Republic of Kazakhstan dated October 31, 2018 No. 600 "On Approval of the Model Rules for Admission to Education in Educational Organizations Implementing Educational Programs of Higher and Postgraduate Education" (hereinafter referred to as the Model Rules).

2. The entrance exam for doctoral studies consists of writing an essay, passing a test for readiness for doctoral studies (hereinafter referred to as TRDS), an exam in the profile of a group of educational programs and an interview.

Block	Points
1. Essay	10
2. Test for readiness for doctoral studies	30
3. Exam according to the profile of the group of the educational program	40
4. Interview	20
Total admission score	100/75

3. The duration of the entrance exam is 4 hours, during which the applicant writes an essay, passes a test for readiness for doctoral studies, and answers an electronic examination. The interview is conducted on the basis of the university separately.

2. Procedure for the entrance examination.

1. Applicants for doctoral studies in the group of educational programs D092 - «Mathematics and statistics» write a problematic / thematic essay. The volume of the essay is at least 250-300 words.

2. The electronic examination card consists of 3 questions.

Topics for exam preparation according to the profile of the group of the educational program.

Discipline **"Mathematical analysis"**

Numerical sequences. Upper and lower limits. Bolzano-Weierstrass theorem and Cauchy criterion for numerical sequences. Limit of functions, continuity and uniform continuity of functions. Weierstrass' theorem on a uniform continuous on a closed segment. Derivative and differential of a function of one variable. The connection between them. Form invariance of the first differential. The concept of inverse function and formulation of the question. Prove the simplest version of the existence theorem for an inverse function. Differentiation of an inverse function of one variable, derivatives of inverse trigonometric functions. Function of several variables. Multiple and repeated limits. The connection between them. Partial derivatives. The differential of a function of several variables. Differentiability of functions of several variables. Differentiation of a complex function of many variables. The concept of an implicit function and the formulation of the question. General theorem on implicit and inverse functions. Jacobian. Change of variables in multiple integral. Green's formula for a double integral. Surface integrals. Basic theorems of integral calculus.

Discipline **"Functional Analysis"**

Metric, normed linear, Banach and Hilbert spaces. Examples of metric, normed, Banach and Hilbert spaces. Sequences and properties of divergent sequences in metric and linear normed spaces. Continuous mappings in metric spaces. Continuity and compactness in metric spaces. The principle of contraction mappings in metric space. General view of a linear bounded functional in a Hilbert space. Riesz's theorem. Measurable sets and their properties. Measurable functions and their properties. Lebesgue integral. Difference between Lebesgue and Riemann integrals. Spaces Lp (Ω) and their properties. Linear operators in Banach and Hilbert spaces. Bounded operators, unbounded operators, closed operators. Operator norm.

Discipline **"Probability theory and stochastic analysis"**

General probabilistic space. Classical and geometric definition of probabilities. Conditional probability. Probability product formula. Independent events, independent trials. Formula of total probability. Bayes' formula. Random variables. Distribution laws of random variables. The mathematical expectation of random variables. Dispersion. Repeated independent tests. Bernoulli's formulas. General definition of a stochastic process and finite-dimensional distributions of a stochastic process. Wiener process. Finite-dimensional distributions of the Wiener process and the characterization property of the Wiener process. Correlation function of a random process. Properties.

Discipline **«Algebra and geometry»**

The concept of algebraic structure. Homomorphisms and isomerphisms of algebraic structures. The group of automorphisms of algebraic structures. Examples. Semigroups. Monoids. Reversible elements. Groups. Cyclic groups. Isomorphisms. Cayley's theorem. Homomorphisms. Kernel and image of a homomorphism. Relationship with normal subgroups. Related classes. Indices. Lagrange's theorem and its consequences. Ring. Zero divisors. Comparisons. Residue classes ring. Ring homomorphisms. Field. Field characteristic. End fields. Construction of the Galois field. Relationships. Equivalence relations, properties of equivalence classes. Partial order relation. Linear order. Smallest, largest, minimum and maximum elements. Prove that a finite partially ordered set always has a minimum element. Dirichlet's principle. Inclusion and exclusion formula. The number of elements in a Cartesian product of a finite number of finite sets.

Discipline **"Differential equations and equations of mathematical physics"**

Theorems of existence and uniqueness of the solution of the Cauchy problem for ordinary differential systems of equations of the first order. Homogeneous linear ordinary differential equation of the n-th order with variable coefficients. Fundamental decision system. Inhomogeneous linear ordinary differential equation of the n-th order with constant coefficients. Systems of homogeneous linear ordinary differential equations, properties of solutions. Ostrogradsky-Liouville formula. Statement of boundary value problems for a linear ordinary differential equation of the second order. Sturm-Liouville problem. Existence and uniqueness theorems for a Sturm-Liouville solution. Existence of eigenvalues of boundary value problems for a linear ordinary differential equation. Definition of the Green's function for the Sturm-Liouville problem and its existence. Solution of boundary value problems for an ordinary differential equation using the Green's function. Inhomogeneous systems of linear differential equations. Method of variation of arbitrary constants (Lagrange method). Classification and reduction to the canonical form of partial differential equations of the second order in the case of many variables. The Cauchy problem for an equation of parabolic type. Fundamental solution of the thermal conductivity operator. Volumetric thermal potential, surface thermal potential and their main properties. The Cauchy problem for an equation of hyperbolic type. The concept of a characteristic for an equation of hyperbolic type. Method of continuation. Statement and basic methods for solving boundary value problems for an elliptic equation. Hadamard's example of the ill-posedness of the Cauchy problem for the Laplace equation. Method of separation of variables. General scheme of the Fourier method. Eigenvalue and eigenfunction problem for the Sturm-Liouville operator. The Fourier method for solving mixed problems for equations of parabolic and hyperbolic types. Cylindrical functions. Bessel's equation. Bessel functions. Problems of Dirichlet and Neumann for the Laplace and Poisson equation. Green's function for the Dirichlet problem, its properties. Solution of a boundary value problem for the Poisson equation using the Green's function.Variation and its properties. The Euler equation. The main lemma of the calculus of variations. The Brachistochron Problem. The simplest problem of the calculus of variations with movable boundaries. Transversality condition. Sufficient conditions for the functional to reach the extremum. Legendre's condition. Variational problems on a conditional extremum. The concept of connections. Reduction to an absolute extremum problem. Lagrange Multipliers. Weierstrass ' theorem in Banach space. The global Minimum theorem. Optimality conditions (in a Banach space). The Lagrange functional. Saddle point.

3. List of references.

Main:

1. V.A. Ilyin, E.G. Poznyak. Foundations of mathematical analysis. Part I. M.: "Science" 1982. 616 p.
2. V.A. Ilyin, E.G. Poznyak. Foundations of mathematical analysis. Part II. M .: "Science" 1980. 447 p.
3. Temirgaliev NT, Mathematics analysis, vol. I-III, 1987,1991 f.
4. Ibrashev Kh.I., Erkegulov Sh.T. Mathematicsқ analysis courses. Almaty. Mektep, Vol. 1.2. 1963-1970.
5. V.A. Zorich, Mathematical Analysis, Part I, II. 2017
6. Kudryavtsev L. D., Course of mathematical analysis, in 3 volumes, 2006.
7. Zorich V. A., Mathematical analysis. Moscow: Nauka, 2007
8. Nauryzbaev .Zh., Nakty analiz, Almaty, “Kazak University”, 2004.
9. Kolmogorov AN, Fomin SV, Elements of the theory of functions and functional analysis, -M.: Nauka, 1989
10. Trenogin V.A. Functional analysis.- Moscow: Nauka, 1967.

11. N.Sh. Kremer. Theory of Probability and Mathematical Statistics. M .: "UNITI", 2000. 544 p.

12. Gnedenko B.V. Course in probability theory and mathematical statistics. - M .: Ed. Moscow State University, 2006.

13. N. Akanbai. Yktimaldyktar theories (I - bulim) - Almaty .: "kazak university", 2001.296 bet.

14. N. Akanbay Yktimaldyktar theory-sons eceptter men zhattygularnyk zhinagy - Almaty: “kazakh university”, 2004. 377 bet.

15. N. Akanbai. Yktimaldyktar theory (3-bөim). Almaty .: "Kazakh university", 2007, 297 bet.

16. N. Akanbai. Yktimaldyktar theory, son eceptters, zhattygularnyk zhinaғy (3-bөlim). Almaty .: "Kazakh University", 2007, 256 bet.

17. Shabat B. V. Introduction to Complex analysis. Part 1. Moscow: Nauka, 1985.

18. Kanguzhin B. E. theory of complex functions of transformation. "No," she said. Kazakh University, 2007.

19. 1.S.A. Badaev. Syzyktyk algebra versus analyticsқ geometry. Volume 2: Syzyқtyқ algebra. Almaty: "LEM Publishing House" ZhShS, 2014. 416 beta.

20. 2.V.A. Ilyin, E.G. Poznyak. Linear algebra. Moscow: "Science" 1984. 294 p.

21. 3. A.I. Kostrikin. Introduction to algebra. Part I. (Fundamentals of algebra). Moscow: Fizmatlit, 2001.254 p.

22. A.I. Kostrikin. Introduction to algebra. Part III. (Basic structures). Moscow: Fizmatlit, 2001.271 p.

23. Isaiah Lankham, Bruno Nachtergaele, Anne Schilling. Linear Algebra As an Introduction to Abstract Mathematics. Copyright c 2007 by the authors. pp. 246.

24. Lyusternik L. A., Sobolev V. I. Short course of functional analysis. - M.: Byssh, shkola, 1982. p. 271.

25. С.А. Бадаев. Сызықтық алгебра және аналитикалық геометрия. Том 1,2: Алматы: LEM, 2014. 416 бет.

26. П.Т. Досанбай. Математикалық логика. Алматы.: «Дәуір» 2011. 280 бет.

27. Suleimenov Zh. Differentsialdyk tendeuler courses, Ouly. Almaty, Kazakh University, 2009.- 440 b.

28. Kadykenov B.M. Differentialdyk tendeulerdin eceptteri men zhattygulary. Almaty, 2002.

29. N.M. Matveev. Integration Methods for Ordinary Differential Equations "4th ed. Minsk:" High School ". 1974.768 p.

30. 4.L.E. Elsgolts. Differential equations and calculus of variations. M .: Science. 1969.425 p.

31. Petrovsky I.G. Lectures on the theory of ordinary differential equations, Moscow, 1970.

32. Pontryagin L.S. Ordinary differential equations. M., 1974.

33. Krasnov M.L., Kiselev A.I., Makarenko G.I. Ordinary differential equations. Tasks and examples with detailed solutions. Moscow: URSS, 2005, 256 p.

34. Tikhonov A.N., Samarsky A.A. Equations of Mathematical Physics. - Moscow: Nauka, 1983.

35. Tokibetov Zh.A., Khairullin E.M. Mathematicalyk physicist tendeuleri, oқulyk. - Astana, Astana polygraphy, 2010.376 b.

36. W.J. Gilbert, W.K. Nickolson. Modern algebra with applications, 2nd ed. Willey, 2004.

Additional:

1. Trenogin V.A., Pisarevsky B.M., Soboleva T.S. Functional analysis tasks and exercises. - M.: Nauka, 1984.

2. Yosida K., Functional analysis. - M.: "Mir", 1967.

3. Kantorovich LV, Akilov GP Functional analysis. - M.: Nauka, 1984.

4. Sadovnichy V.A. Operator theory. -M. "High School", 2000.
5. Natanson IP, Theory of functions of a real variable, Moscow: Gostekhizdat, 1957.
6. Sevastyanov B.A. Theory of Probability and Mathematical Statistics. M.: "Science", 1982.256 p.,
7. Gnedenko B.V. Course in probability theory and mathematical statistics. M .: "UNINTI", 1988.448 p.,
8. Agapov G.I. Probability Theory Problem Book. M.: "High school", 1985. 112 p.
9.V.A. Kolemaev, O.V. Staroverov, V.B. Turundaevsky Probability Theory and Mathematical Statistics - Moscow: "Higher School", 1991. 400 p.
10. N. Akanbai, Z.I. Suleimenova, S.Қ. Tepeeva Yktimaldyktar theories of women mathematicians statisticdan test of suraktary, Almaty, “Kazakh university”, 2005 f., 254 bet.
11. Vladimirov V.S., Zharinov V.V. Equations of mathematical physics: Textbook for universities. 2nd ed. - M.: Fizmatlit, 2003.
12. Khompysh H. Mathematics and Physics Tedeuleri. Oқu құraly. -Almaty: Kazakh university, 2017
13. Krasnov, M.L. Ordinary differential equations, Moscow: URSS, 2002, 253 p.
14. Fedoryuk, M.V. Ordinary differential equations: Ed. 3rd, ster. - SPb.: Lan, 2003. - 447 p.
15. Filippov, A.F. Collection of problems on differential equations: Ed. 2nd.- Moscow: Publishing house of LCI, 2008.- 235 p.
16.V.A. Ilyin, E.G. Poznyak. Linear algebra. Moscow: "Science" 1984. 294 p.
17. V.A. Ilyin, E.G. Poznyak. Analytic geometry. Moscow: Nauka 1971. 232 pp.
18. Taimanov I. A., Lectures on Differential Geometry, 2002.
19. Krasnov M. L., Kiselev A. I., Makarenko G. I. Ordinary differential equations (problems and examples with detailed solutions). - M. 2002, p. 176.
20. Gelfand I. M., Fomin S. V. Variational calculus – - M., 1961
21. Aysagaliev S. A. Lectures on optimal control. - Almaty, 2007.
22. Vasiliev F. P. Lectures on methods for solving extreme problems. - M., 1974.